# ACCELERATED METHODS FOR ESTIMATING THE DURABILITY OF PLAIN BEARINGS

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#### ABSTRACT

The paper presents methods for determining the durability of slide bearings. The developed methods enhance the calculation process by even 100000 times, compared to the accurate solution obtained with the generalized cumulative model of wear. The paper determines the accuracy of results for estimating the durability of bearings depending on the size of blocks of constant conditions of contact interaction between the shaft with small out-of-roundedness and the bush with a circular contour. The paper gives an approximate dependence for determining accurate durability using either a more accurate or an additional method.

**Keywords:** enhanced numerical methods, durability evaluation, plain bearings, accuracy deviations.

#### INTRODUCTION

Plain bearings find numerous applications in machine design as well as in various devices that facilitate human life. The literature on the subject provides simplified methods [1-14] for estimating the durability of these bearings where it is assumed that the shaft and bush have an ideal (i.e. circular) section of contours. However, in reality these components of slide bearings are produced with out-of-roundness deviations (ovality, trilobing, tetralobing) that have – as demonstrated in [15, 16] – an effect on the durability of bearings.

The kinematics of wear of the shaft with outof-roundedness can be investigated using either the cumulative model of wear [15–17] or the generalized cumulative model of wear [18] for the interval-discrete interaction between bearing components. The cumulative model of wear allows estimating the degree of wear and durability of a bearing for single-area contact of the components, while the generalized cumulative model of wear is applied to examine mixed contact.

The above models entail dividing the contour of the shaft with out-of-roundedness into discretization intervals  $\Delta \alpha_2$ , where both initial con-

tact pressures and contact area are assumed to be constant. In a full rotation of the shaft with outof-roundedness, there is *j*-th interaction with the bush ( $j = 360^{\circ}/\Delta \alpha_2$ ).

Bearing components undergo wear during rotation, hence the initial contact parameters change and their wear is cumulative.

With application of the interval-discrete pattern of tribocontact interaction, the time of computations is considerably long, particularly at low values of the discretization interval  $\Delta \alpha$ , of shaft contour that ensure the most accurate numerical solution of the problem. An increase in  $\Delta \alpha_2$  leads to a proportional decrease in the estimated durability of a bearing. Nonetheless, the computation time still remains considerably long. In order to decrease effectively the time of computations, the express method [19] with an interval-rangeblock calculation pattern has been developed. The method is based on the assumption that parameters of contact interaction for particular intervals of discretization remain constant during a certain number of shaft rotations (block size). Such an approach ensures a shorter time of computations, proportionally to the size of a block with constant conditions of interaction between the bearing components. The present paper presents the results of this problem as solved by the express method using various calculation methods.

# **CALCULATION METHODS**

The slide bearing (Fig. 1) carries the load N, while shaft 2 rotates with the angular velocity  $\omega_2$ . The bearing has a radial clearance of  $\varepsilon = R_1 - R_2$ , where  $2R_1$  denotes the nominal diameter of the bush and  $2R_2$  denotes the diameter of the shaft. Under the action of load, the bearing components contact each other in the  $2R_2\alpha_0$  area where contact pressures  $p(\alpha)$  are generated. The bearing is made of different materials, whose properties and resistance to wear differ.



Fig. 1. Schematic diagram of a slide bearing

Due to the fact that the components of bearing exhibit low ovality, they produce either single-area (Fig. 2a) or double-area (Fig. 2b) contact. During a rotation of the shaft ( $\alpha_2$ >0), two types of interactions are possible: full single-area and mixed (single – double – single-area). The interaction between bearing components with the contour ovality  $L_1$  and  $L_2$  is described by the following contact parameters:

- single area contact (Fig. 2a):  $2\alpha_{0\delta}(\alpha_2)$  variable contact angle, contact pressures  $p(\alpha_2, \delta)$ , maximum contact pressures  $p(0, \delta)$ , contact area  $W = 2R_2\alpha_{0\delta}$ ;
- mixed contact (Fig. 2b):  $2\gamma$  symmetric contact angle or asymmetric contact angles  $2\gamma_1(\alpha_2)$ ,  $2\gamma_2(\alpha_2)$ ; symmetric contact pressures  $p(\gamma, \delta)$  and asymmetric contact pressures  $p(\alpha_2, \delta)$ ; symmetric contact area  $W_{1,2} = 2R_2\gamma$  and asymmetric contact areas  $W_1 = 2R_2\gamma_1$ ,  $W_2 = 2R_2\gamma_2$ .

The contact angle  $2\lambda$  of bearing components is determined by the method discussed in the study [20]. In the case of the symmetric double-area contact (Fig. 2b), the forces  $N_1 = N_2 = N/(2\cos\lambda)$ .

However, for the asymmetric contact  $N_1 \neq N_2$ ,  $\lambda_1 \neq \lambda_2$ ,  $2\lambda_1 \neq 2\lambda_2$ ,  $p(\lambda_1, \delta) \neq p(\lambda_2, \delta)$  [17]. The ovality of contours of the bearing components is described by the deviations  $\delta_1 = R_1 - R_1'$ ,  $\delta_2 = R_2' - R_2$  (Fig. 2).

a)

b)



Fig. 2. Single-area (a) and double-area (b) contact of shaft and bush

Using the generalized cumulative model of wear [18], a numerical calculation was performed for the problem of full single-area contact and mixed contact of the bearing components, where the following parameters were applied: N=0.1 MN;  $R_2 = 50$  mm; v = 0.0628 m/sec – sliding velocity; f = 0.04 – sliding friction factor at boundary friction;  $\varepsilon = 0.21$ ; 0.41 mm;  $\delta_1 = 0$  mm;  $\delta_2 = 0$ ; 0.05; 0.1; 0.15; 0.2; 0.3; 0.4 mm;  $\delta_1 + \delta_2 \le \varepsilon$ ;  $n_2 = 12$  rpm – number of rotations of the shaft;  $h_{1*} = 0.3$  mm – maximum wear of the bush; the shaft

is made of steel (hardened + tempered), while the bush is made of bronze CuSn5Zn5Pb5;  $\Delta \alpha_2 = 10^{\circ}$  – the discretization interval of the shaft contour; B = 72000, 7200, 720, 12, 1 rotations – block sizes.

# RESULTS

The results of solving the problem are listed in:

- Tables 1, 2 the accurate method based on the interval-block pattern (B = 1 rotation);
- Tables 3, 4 (numerator) the approximate method based on the interval-block pattern (*B* = 12, 720, 7200, 72 000 rotations);

**Table 1.** Bearing durability  $n_{2*}$  as calculated by the accurate method ( $\varepsilon = 0.21 \text{ mm}$ )

ā mm	<i>B</i> = 1 rot			
0 <sub>2</sub> , 11111	n <sub>2*</sub> , rot	$h_{\! m l^*}$ , mm		
0.2	1 722 413	0.3		
0.15	1 663 970	0.3		
0.1	1 651 362	0.3		
0.05	1 413 160	0.3		
0.0	1 332 336	0.3		

• Tables 3, 4 (denominator) – the more accurate method based on the results of the approximate solution.

An increase in radial clearance leads to decreased durability of the bearing.

Here, the actual wear  $h_1$  of the bush is always somewhat higher than the maximum wear  $h_{1*}$ (Tables 3, 4).

The bearing durability  $h_{1*}$  was estimated more accurately based on its approximate durability  $n_2$ and the corresponding wear  $h_1 > h_{1*}$  of the bush in the following way:  $n_{2*} = n_2(h_{1*}/h_1)$ . Tables 5 and 6 give the deviations  $\Delta_B$  of the solutions.

**Table 2.** Bearing durability  $n_{2*}$  as calculated by the accurate method ( $\varepsilon = 0.41 \text{ mm}$ )

δ	<i>B</i> = 1 rot		
0 <sub>2</sub> , mm	n <sub>2*</sub> , rot	<i>h</i> ₁∗, mm	
0.4	1 239 410	0.3	
0.3	1 195 707	0.3	
0.2	1 226 752	0.3	
0.1	1 047 977	0.3	
0	982 501	0.3	

**Table 3.** Bearing durability  $n_{2*}$  calculated by the approximate method ( $\varepsilon = 0.21$  mm)

δ		<i>B</i> = 72 000 rot		
0 <sub>2</sub> , mm	n <sub>2</sub> /n <sub>2*</sub>	h <sub>1</sub> /h <sub>1*</sub>	$\Delta, \%$	
0.0	1 656 000	0.30098	0.22	
0.2	1 650 535	0.3	0.33	
0.15	1 656 000	0.31155	2.95	
0.15	1 592 244	0.3	3.65	
0.1	1 584 000	0.30110	0.27	
0.1	1 578 139	0.3	0.37	
0.05	1 368 000	0.30570	1.0	
0.05	1 342 008	0.3	1.9	
0	1 296 000	0.31040	2 47	
0	1 251 029	0.3	5.47	
δ mm	<i>B</i> = 7200 rot			
0 <sub>2</sub> , mm	$n_{2}/n_{2^{\star}}$	h <sub>1</sub> /h <sub>1*</sub>	Δ, %	
0.2	1 720 800	0.30098	0.33	
0.2	1 715 121	0.3	0.55	
0.15	1 663 200	0.30116	0.30	
0.15	1 691 205	0.3	0.59	
0.1	1 648 800	0.30110	0.37	
0.1	1 642 700	0.3	0.37	
0.05	1 411 200	0.30110	0.37	
0.05	1 405 979	0.3	0.57	
0	1 324 800	0.30048	0.16	
0			0.10	

ō mm		<i>B</i> = 720 rot		
0 <sub>2</sub> , mm	n <sub>2</sub> /n <sub>2*</sub>	h <sub>1</sub> /h <sub>1*</sub>	Δ, %	
	1 722 240	0.3001	0.02	
0.2	1 721 723	0.3	0.03	
0.15	1 663 920	0.30012	0.04	
0.15	1 663 255	0.3	0.04	
0.1	1 650 960	0.3001	0.02	
0.1	1 650 415	0.3	0.03	
0.05	1 412 410	0.3000	0.00	
0.05	1 412 410	0.3	0.00	
0	1 332 000	0.3001	0.03	
0	1 331 560 0.3 0.03			
ōmm	<i>B</i> = 12 rot			
0 <sub>2</sub> , mm	n <sub>2</sub> /n <sub>2*</sub>	h <sub>1</sub> /h <sub>1*</sub>	Δ, %	
0.2	1 722 396	0.3000	0.00	
0.2	1 722 396	0.3	0.00	
0.15	1 663 944	0.3000	0.00	
0.15	1 663 944	0.3	0.00	
0.1	1 651 348	0.3000		
0.1	1 651 348	0.3	0.00	
0.05	1 413 156	0.3000	0.00	
0.05	1 413 156	0.3	0.00	
0	1 332 320	0.3000	0.00	

ā mm	<i>B</i> = 72 000 rot			
0 <sub>2</sub> , IIIII	n <sub>2</sub> /n <sub>2*</sub>	$h_{1}/h_{1*}$	Δ, %	
0.4	1 224 000	0.313 110	4.07	
0.4	1 170 499	0.3	4.37	
0.2	1 152 000	0.306 900	2.24	
0.3	1 125 399	0.3	2.31	
0.2	1 224 000	0.317 040	5.68	
0.2	1 154 477	0.3		
0.1	1 008 000	0.309 107	2.04	
0.1	977 400	0.3	3.04	
0	936 000	0.307 786	2.60	
0	911 708	0.3	2.00	

**Table 4.** Bearing durability  $n_{2*}$  calculated by the approximate method ( $\varepsilon = 0.41$  mm)

ō mm	<i>B</i> = 720 rot			
0 <sub>2</sub> , 11111	n <sub>2</sub> /n <sub>2*</sub>	h <sub>1</sub> /h <sub>1*</sub>	Δ, %	
0.4	1 239 120	0.300 096	0.03	
0.4	1 238 724	0.3	0.03	
0.2	1 195 200	0.300 041	0.01	
0.3	1 195 037	0.3	0.01	
0.2	1 226 160	0.300 132	0.04	
0.2	1 225 746	0.3	0.04	
0.1	1 047 600	0.300 098	0.02	
0.1	1 047 258	0.3	0.03	
0	982 080	0.300 092	0.02	
	981 779	0.3	0.03	

The analysis of the obtained results demonstrates that an increase in block size leads to higher deviations  $\Delta_B$ . At radial clearance  $\varepsilon = 0.21$  mm (Table 5), for B = 720 rotations, the estimated durability decreases by 0.04–0.058% compared to the accurate durability (B = 1 rot); for B =7200 rotations it decreases by 0.42–0.730%; for B = 72000 rotations it decreases by 4.17–6.1% depending on shaft ovality value. In the case of shafts with a circular section ( $\delta_2 = 0$ ), the deviations will be the highest. If radial clearances increase, the deviation  $\Delta_B$  increases as well, as shown in Table 6 for  $\varepsilon = 0.41$  mm. It is assumed that block B = 7200 rotations will be optimum for the discretization interval  $\Delta \alpha_2 = 10^0$ ;

Table 7 list an additive method based on the adjusted interval-block pattern (B =  $72000 \rightarrow 7200 \rightarrow 12 \rightarrow 1 \text{ rot}$ ), using the formula:

$$n_{2^*} = x_1 B_{72000} + x_2 B_{7200} + x_3 B_{720} + x_4 B_{12} + x_5 B_1, \quad (1)$$

where:  $x_1$  is the maximum number of basic blocks  $B_{p^2}$  for which  $h_1 < h_{1*}$ ;

 $x_{2}^{P}$ ,  $x_{3}$ ,  $x_{4}$  are the respectively maximum number of successive block sizes which

δ <sub>2</sub> ,	<i>B</i> = 7200 rot			
mm	n <sub>2</sub> /n <sub>2*</sub>	h <sub>1</sub> /h <sub>1*</sub>	Δ, %	
0.4	1 238 400	0.300 932	0.21	
0.4	1 234 561	0.3	0.51	
0.2	1 195 200	0.301 104	0.27	
0.5	1 190 778	0.3	0.37	
0.2	1 224 000	0.301 188	0.40	
0.2	1 219 153	0.3		
0.1	1 044 000	0.300 923	0.31	
0.1	1 040 788	0.3		
0	979 200	0.301 191	0.40	
0	975 313	0.3	0.40	

5 mm	<i>B</i> = 12 rot			
0 <sub>2</sub> , mm	n <sub>2</sub> /n <sub>2*</sub>	h <sub>1</sub> /h <sub>1*</sub>	Δ, %	
0.4	1 239 408	0.300 001	0.00	
0.4	1 239 403	0.3	0.00	
0.2	1 195 700	0.300 000	0.00	
0.3	1 195 697	0.3	0.00	
0.2	1 226 611	0.300 003	0.00	
0.2	1 226 000	0.3	0.00	
0.1	1 047 970	0.300 002	0.00	
	1 047 963	0.3	0.00	
0	982 500	0.300 003	0.00	
	982 489	0.3	0.00	

will satisfy a given condition;

 $x_5$  is the number of blocks B = 1 rotation that ensure obtaining accurate value of wear  $h_{1*}$ .

The results of investigating the effect of the size of blocks of constant conditions of contact on the durability of a bearing with shaft ovality demonstrate that the deviations from the accurate result caused by blocks with a size of up to 10000 rotations do not exceed 1%, which is also the number of times by which the computation time is reduced.

For practical reasons, the additive method based on the adjusted interval – block pattern is most rational to apply to solve the problem. Apart from a much simpler and faster procedure of numerical calculation, the accuracy of results obtained is similar to the results obtained with a more accurate method (Tables 5, 6). Analyzing the durability estimation results (Tables 5–7), it is found that the durability  $n_{2*}^{(D)}$  of the accurate solution can be determined based on the durability  $n_{2*}^{(Dk)}$  or  $n_{2*}^{(d)}$  in the following way:

$\delta_2^{}$ , mm	<i>B</i> , rot	<i>n</i> <sub>2*</sub> , rot	$\Delta_{\!_B},\%$
	1	1 722 413	0.000
	12	1 722 396	0.001
0.2	720	1 721 723	0.040
	7200	1 715 121	0.423
	72 000	1 650 535	4.173
	1	1 663 970	0.000
	12	1 663 944	0.001
0.15	720	1 663 255	0.032
	7200	1 656 714	0.436
	72 000	1 592 244	4.311
	1	1 651 362	0.000
	12	1 651 348	0.001
0.1	720	1 650 415	0.057
	7200	1 642 700	0.525
	72 000	1 578 139	4.434
	1	1 413 160	0.000
	12	1 413 156	0.001
0.05	720	1 412 410	0.053
	7200	1 405 979	0.508
	72 000	1 342 008	5.035
	1	1 332 336	0.000
	12	1 332 320	0.001
0	720	1 331 560	0.058
	7200	1 322 680	0.725
	72 000	1 251 029	6.103

**Table 5.** Deviations of the approximate durability with regard to the accurate one ( $\varepsilon = 0.21$  mm)

**Table 6.** Deviations of the approximate durability with regard to the accurate one ( $\varepsilon = 0.41$  mm).

<sub>2</sub> , mm	B, rot	n <sub>2*</sub> , rot	$\Delta_{\!_B},\%$
	1	1 239 410	0.000
	12	1 239 396	0.001
0.4	720	1 238 729	0.055
	7200	1 234 561	0.398
	72 000	1 170 499	5.56
	1	1 195 707	0.000
	12	1 195 697	0.001
0.3	720	1 195 037	0.056
	7200	1 190 778	0.41
	72 000	1 125 399	5.88
	1	1 226 752	0.000
	12	1 226 700	0.004
0.2	720	1 225 746	0.082
	7200	1 219 153	0.619
	72 000	1 154 477	5.891
	1	1 047 977	0.000
	12	1 047 963	0.001
0.1	720	1 047 258	0.069
	7200	1 040 788	0.681
	72 000	977 400	6.731
	1	982 501	0.000
	12	982 500	0.000
0	720	981 779	0.074
	7200	975 313	0.732
	72 000	911 708	7.205

**Table 7.** Bearing durability for different sizes of basic blocks ( $\varepsilon = 0.41 \text{ mm}$ )

S	<i>B</i> = 1 rot	<i>B<sub>p</sub></i> = 7200 rot		<i>B<sub>p</sub></i> = 72 000 rot	
0 <sub>2</sub> , 11111	n <sub>2*</sub> , rot	<i>n</i> <sub>2*</sub> , rot	$\Delta_{\Sigma}$ ,%	n <sub>2*</sub> , rot	Δ <sub>Σ</sub> , %
0.4	1 239 410	1 232 469	0.56	1 170 375	5.57
0.3	1 195 707	1 189 011	0.56	1 125 160	5.90
0.2	1 226 752	1 219 141	0.62	1 154 341	5.90
0.1	1 047 977	1 040 733	0.69	975 973	6.87
0	982 501	975 301	0.73	910 501	7.33

**Note:**  $\Delta_{\Sigma}$  – deviations of the additive durability from the accurate durability (*B* = 1 rotation).

$$n_{2*}^{(D)} \approx n_{2*}^{(Dk)} + B \text{ or } n_{2*}^{(D)} \approx n_{2*}^{(A)} + B_p,$$
 (2)

where: (*Dk*) and (*A*) denote, respectively, the more accurate and additive methods for solving the problem.

Tables 8 and 9 list durability differences  $\Delta n_2 = n_{2*}^{(D)} - n_{2*}^{(Dk)}, n_{2*}^{(A)}$  which confirm the formula (2).

### **CONCLUSIONS**

- 1. Values of deviations of durability determined by the more accurate and additive methods were compared to those obtained by the accurate method.
- 2. It is found that the computation time decreases es directly proportional to the size of blocks of constant conditions of interaction.

**Table 8.** Differences  $\Delta n_2 = n_{2*}^{(D)} - n_{2*}^{(A)}$  for the durability *D* and *Dk* ( $\varepsilon = 0.41$  mm)

$\delta_2^{}$ , mm	B, rot	n <sub>2*</sub> , rot	$\Delta n_2$ , rot
	1	1 239 410	-
	12	1 239 396	14
0.4	720	1 238 729	681
	7200	1 234 561	4849
	72 000	1 170 499	68 911
	1	1 195 707	-
	12	1 195 697	10
0.3	720	1 195 037	670
	7200	1 190 778	4929
	72 000	1 125 399	70 308
	1	1 226 752	-
	12	1 226 700	52
0.2	720	1 225 746	1006
	7200	1 219 153	7599
	72 000	1 154 477	72 275
	1	1 047 977	-
	12	1 047 963	14
0.1	720	1 047 258	719
	7200	1 040 788	7189
	72 000	977 400	70 577
	1	982 501	-
	12	982 500	1
0	720	981 779	722
	7200	975 313	7188
	72 000	911 708	70 793

- 3. We have got an approximate dependence for estimating accurate durability of a bearing based on the more accurate and additive method.
- 4. It is found that the additive method is a simple and effective way of solving the problem of estimating the durability of a slide bearing.

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δ <sub>2</sub> , mm	<i>B</i> = 1 rot	<i>B<sub>p</sub></i> = 7200 rot		B <sub>p</sub> = 72 000 rot	
	n <sub>2*</sub> , rot	<i>n</i> <sub>2*</sub> , rot	$\Delta n_2^2$ , rot	n <sub>2*</sub> , rot	$\Delta n_2^2$ , rot
0.4	1 239 410	1 232 469	6941	1 170 375	69 035
0.3	1 195 707	1 189 011	6696	1 125 160	70 547
0.2	1 226 752	1 219 141	7611	1 154 341	72 411
0.1	1 047 977	1 040 733	7244	975 973	72 004
0	982 501	975 301	7200	910 501	72 000

**Table 9.** Differences  $\Delta n_2 = n_{2*}^{(D)} - n_{2*}^{(A)}$  for the durability *D* and *A* ( $\varepsilon = 0.41$  mm)

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